

Test Assignment 3

① Given curve $y = 3x^2 - 4x + 5$. Let $x_1 = 1$.

$$\Rightarrow \frac{dy}{dx} = 6x - 4$$

At $x_1 = 1$, the slope is

$$m = \frac{dy}{dx}(1) = 6(1) - 4 = 2$$

$$\text{And } y_1 = 3x_1^2 - 4x_1 + 5 = 3(1)^2 - 4(1) + 5 = 4$$

$\therefore$ Equation of tangent line is

$$y - y_1 = m(x - x_1)$$

$$\Rightarrow y - 4 = 2(x - 1) = 2x - 2$$

$$\Rightarrow y = 2x - 2 + 4$$

$$\Rightarrow y = 2x + 2$$

② Given $q(w) = 10w + \sqrt{w}$; $0 \leq w \leq 25$
and $R(q) = 20q - 0.02q^2$; $0 \leq q \leq 255$

$$(i) \text{ We have } \frac{dq}{dw} = 10 + \frac{1}{2\sqrt{w}}$$

$$\text{and } \frac{dR}{dq} = 20 - 0.04q$$

$$\text{so } \frac{dR}{dw} = \frac{dR}{dq} \cdot \frac{dq}{dw} = (20 - 0.04q) \left(10 + \frac{1}{2\sqrt{w}} \right)$$

$$(ii) \text{ when } w = 16, \quad q(w) = (10)(16) + \sqrt{16} = 164 \text{ m}^2 \text{ of carpet}$$

$$\begin{aligned} (iii) \quad \frac{dR}{dw}(q=164, w=16) &= (20 - 0.04(164)) \left(10 + \frac{1}{2\sqrt{16}} \right) \\ &= (13.44)(10.125) \\ &= \$136.08 \end{aligned}$$

Revenue will increase by approximately \$136.08
if an additional worker is hired.

③ Given the supply equation

$$20p^2 - q^2 = 8pq$$

$$\Rightarrow 40p \frac{dp}{dt} - 2q \frac{dq}{dt} = 8 \left(q \frac{dp}{dt} + p \frac{dq}{dt} \right)$$

We are told that $q=10$, $p=\$5$ and $\frac{dp}{dt} = \$0.50$.

so plugging in

$$(40)(5)(0.5) - 2(10) \frac{dq}{dt} = 8 \left((10)(0.5) + (5) \left(\frac{dq}{dt} \right) \right)$$

$$\Rightarrow 100 - 20 \frac{dq}{dt} = 8 \left(5 + 5 \frac{dq}{dt} \right) = 40 + 40 \frac{dq}{dt}$$

$$\Rightarrow 40 \frac{dq}{dt} + 20 \frac{dq}{dt} = 100 - 40$$

$$\Rightarrow 60 \frac{dq}{dt} = 60$$

$$\Rightarrow \frac{dq}{dt} = \frac{60}{60} = 1$$

$\therefore$ The factory is willing to produce one more calculator if retailers increase the price by 50 cents.

④ Given $K(I) = -0.006I^2 + 0.75I - 0.05\sqrt{I}$
and $I = 25$

(i) Christine's marginal propensity to consume is

$$\begin{aligned} \frac{dK}{dI} &= -0.012I + 0.75 - 0.05 \left(\frac{1}{2\sqrt{I}} \right) \\ &= -0.012I + 0.75 - \frac{0.025}{\sqrt{I}} \end{aligned}$$

(ii) when $I=25$,

$$\frac{dK}{dI}(25) = -0.012(25) + 0.75 - \frac{0.025}{\sqrt{25}} = 0.445 = 44.5\%$$

Christine is likely to spend 44.5%

and save $100\% - 44.5\% = 55.5\%$ of her bonus.

(iii) Christine will spend $\$1,500 \times 44.5\% = \$667.50$

and save $\$1,500 \times 55.5\% = \$832.50$